NAG Fortran Library Routine Document

F07JEF (DPTTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07JEF (DPTTRS) computes the solution to a real system of linear equations $AX = B$, where A is an n by n symmetric positive-definite tridiagonal matrix and X and B are n by r matrices, using the LDL^{T} factorization returned by F07JDF (DPTTRF).

2 Specification

```
SUBROUTINE F07JEF (N, NRHS, D, E, B, LDB, INFO)
INTEGER              N, NRHS, LDB, INFO
double precision     D(*), E(*), B(LDB,*)
```

The routine may be called by its LAPACK name ***dpttrs***.

3 Description

F07JEF (DPTTRS) should be preceded by a call to F07JDF (DPTTRF), which computes a modified Cholesky factorization of the matrix A as

$$A = LDL^{\mathrm{T}},$$

where L is a unit lower bidiagonal matrix and D is a diagonal matrix, with positive diagonal elements. F07JEF (DPTTRS) then utilizes the factorization to solve the required equations. Note that the factorization may also be regarded as having the form $U^{\mathrm{T}}DU$, where U is a unit upper bidiagonal matrix.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

5 Parameters

1: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

2: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathrm{NRHS} \geq 0$.

3: D($*$) – ***double precision*** array *Input*

Note: the dimension of the array D must be at least $\max(1, \mathrm{N})$.

On entry: must contain the n diagonal elements of the diagonal matrix D from the LDL^{T} factorization of A.

4: E($*$) – ***double precision*** array *Input*

Note: the dimension of the array E must be at least $\max(1, N - 1)$.

On entry: must contain the $(n-1)$ subdiagonal elements of the unit lower bidiagonal matrix L. E can also be regarded as the superdiagonal of the unit upper bidiagonal matrix U from the $U^{T}DU$ factorization of A.

5: B(LDB,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, NRHS)$.

On entry: the n by r matrix of right-hand sides B.

On exit: the n by r solution matrix X.

6: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07JEF (DPTTRS) is called.

Constraint: LDB $\geq \max(1, N)$.

7: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1\|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Following the use of this routine F07JGF (DPTCON) can be used to estimate the condition number of A and F07JHF (DPTRFS) can be used to obtain approximate error bounds.

8 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ is proportional to nr.

The complex analogue of this routine is F07JSF (ZPTTRS).

9 Example

This example solves the equations

$$AX = B,$$

where A is the symmetric positive-definite tridiagonal matrix

$$A = \begin{pmatrix} 4.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 10.0 & -6.0 & 0 & 0 \\ 0 & -6.0 & 29.0 & 15.0 & 0 \\ 0 & 0 & 15.0 & 25.0 & 8.0 \\ 0 & 0 & 0 & 8.0 & 5.0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 6.0 & 10.0 \\ 9.0 & 4.0 \\ 2.0 & 9.0 \\ 14.0 & 65.0 \\ 7.0 & 23.0 \end{pmatrix}.$$

9.1 Program Text

```
*     F07JEF Example Program Text
*     Mark 21 Release. NAG Copyright 2004.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, NRHSMX
      PARAMETER        (NMAX=50,NRHSMX=4)
      INTEGER          LDB
      PARAMETER        (LDB=NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N, NRHS
*     .. Local Arrays ..
      DOUBLE PRECISION B(LDB,NRHSMX), D(NMAX), E(NMAX-1)
*     .. External Subroutines ..
      EXTERNAL         DPTTRF, DPTTRS, X04CAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07JEF Example Program Results'
      WRITE (NOUT,*)
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, NRHS
      IF (N.LE.NMAX .AND. NRHS.LE.NRHSMX) THEN
*
*        Read the upper bidiagonal part of the tridiagonal matrix A from
*        data file
*
         READ (NIN,*) (E(I),I=1,N-1)
         READ (NIN,*) (D(I),I=1,N)
*
*        Read the right hand matrix B
*
         READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
*        Factorize the tridiagonal matrix A
*
         CALL DPTTRF(N,D,E,INFO)
*
         IF (INFO.EQ.0) THEN
*
*           Solve the equations AX = B
*
            CALL DPTTRS(N,NRHS,D,E,B,LDB,INFO)
*
*           Print the solution
*
            IFAIL = 0
            CALL X04CAF('General',' ',N,NRHS,B,LDB,'Solution(s)',IFAIL)
*
         ELSE
            WRITE (NOUT,99999) 'The leading minor of order ', INFO,
     +        ' is not positive definite'
         END IF
      ELSE
         WRITE (NOUT,*) 'NMAX and/or NRHSMX too small'
```

```
      END IF
      STOP
*
99999 FORMAT (1X,A,I3,A)
      END
```

9.2 Program Data

```
F07JEF Example Program Data
  5     2                     :Values of N and NRHS
       -2.0  -6.0  15.0   8.0 :End of super-diagonal E
  4.0  10.0  29.0  25.0   5.0 :End of diagonal D
  6.0  10.0
  9.0   4.0
  2.0   9.0
 14.0  65.0
  7.0  23.0                   :End of matrix B
```

9.3 Program Results

```
 F07JEF Example Program Results

 Solution(s)
             1          2
 1      2.5000     2.0000
 2      2.0000    -1.0000
 3      1.0000    -3.0000
 4     -1.0000     6.0000
 5      3.0000    -5.0000
```